Singularity and entropy in the bulk viscosity dark energy model

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Abstract

In this paper bulk viscosity is introduced to describe the effects of cosmic non-perfect fluid on the cosmos evolution and to build the unified dark energy (DE) with (dark) matter models. Also we derive a general relation between the bulk viscosity form and Hubble parameter that can provide a procedure for the viscosity DE model building. Especially, a redshift dependent viscosity parameter $\zeta \propto \lambda_0 + \lambda_1(1+z)^n$ proposed in the previous work by X.H.Meng and X.Dou in 2009[17] is investigated extensively in this present work. Further more we use the recently released supernova dataset (the Constitution dataset) to constrain the model parameters. In order to differentiate the proposed concrete dark energy models from the well known ΛCDM model, statefinder diagnostic method is applied to this bulk viscosity model, as a complementary to the Om parameter diagnostic and the deceleration parameter analysis performed by us before. The DE model evolution behavior and tendency are shown in the plane of the statefinder diagnostic parameter pair $\{r, s\}$ where the fixed point represents the ΛCDM model. The possible singularity property in this bulk viscosity cosmology is also discussed to which we can conclude that in the different parameter regions chosen properly, this concrete viscosity DE model can have various late evolution behaviors and the late time singularity could be avoided. We also calculate the cosmic entropy in the bulk viscosity dark energy frame, and find that the total entropy in the viscosity DE model increases monotonously with respect to the scale factor evolution, thus this monotonous increasing property can indicate an arrow of time in the universe evolution, though the quantum version of the arrow of time is still puzzling.

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I. INTRODUCTION

Type Ia supernova and other astrophysics observations together indicate that our universe is accelerating now [1]. Different models are proposed to try to describe or understand this surprisingly exotic phenomenon. If we make the assumption that general relativity is still correct to the scale of cosmos, an effective term contributes to negative pressure should be added to the right hand side of Einstein's field equation in the general theory of relativity to explain the recent stage speed-up of our observational universe expansion. This is the basic idea to the so called dark energy concept. So far for the ten years' old DE phenomena there are many both theoretical and observational attempts to understand the mechanism. The introduction of the cosmological constant corresponds to a negative pressure fluid with a specially constant density all the time which has been playing a key role in the universe evolution by uniformly distributed over the whole cosmic space-time and media, but the cosmological constant existence raises serious fundamental physics problem, the so-called cosmological constant problem for both new (coincidence) and old (so tiny) ones. Another class of models try to modify the traditional Einstein's general theory of relativity to the large cosmology scale, by arguing that the recently appearing acceleration phase of the unverse expansion comes from the break down of general relativity in cosmic scale. The so-called $f(R)$ gravity, to mention one for example, which generalizes the Hilbert-Einstein action is categorized in this class [2]. (For more details and references, you may see the recent reviews [3] [4]).

In the context of perfect fluid, some models based on fluid mechanics method are studied extensively, such as the Chaplygin gas and generalized Chaplygin gas models which modify the equation of state, and barotropic fluids dark Energy [5]. Also for the purpose to consider more realistic situation in the evolution of the universe, the concept of viscosity is introduced into the investigation of the cosmos evolution from fluid mechanics [6] [7] [8] [9] [10] [11] [12]. Earlier attempt [13] in this area even "predicts" the late acceleration of the universe expansion. The dissipative effect in the fluid is always due to shear and bulk viscosity characterized by shear viscosity parameter η and bulk viscosity parameter ζ. In the study of cosmology, the shear viscosity disappears in the Friedmann-Robertson-Walker's isotropic and homogeneous framework with the largest spherical symmetry. Only bulk viscosity could play a role in the realistic models. Its effects can be shown from an added correction term with the minus sign to cosmic pressure $\tilde{p} = p - 3\zeta H$. This composite formula motives the trial on the connection between bulk viscosity of the cosmic fluid and dark components (energy and matter). As we understand so far that purely gravitational probes can only provide information on a single effective matter-energy fluid, which may consist of the dark energy and (dark) matter composites as well as the least dominated radiation energy at present universe, so for the dominated two functioning dark components we can describe them as a single dark fluid [14]. In this paper, we discuss a concrete unified model building by paying attention on the modification of the cosmic media from the simple assumption as a perfect fluid to a piratical viscous fluid encoded in the energy-stress tensor contents, that is, at the right hand side of the Einstein's field equation. For the bulk viscosity DE model building a remark is needed here. Considering the observational dark energy ingredient fraction today, it takes about $\frac{2}{3}$ of the whole matter-energy density, so the

corresponding pressure provided by the bulk viscosity correction dominantly surpasses the remaining pressure contributions from other cosmic energy-matters. This is obviously different from the traditional non-equilibrium thermodynamics, in which the viscosity contribution is only a little correction to the pressure term. Some researches try to find a mechanism to support such a fluid behavior by a kind of non-standard interaction introduced between the dark matter and energy components [18] [19]. It is certainly important for these attempts to find more support or hints behind cosmic observations in the study of bulk viscosity cosmology along this possible line.

For concrete model building, detailed form of viscosity parameter is needed to settle down the theoretical framework. In the reference [14], a scale factor dependent viscosity is proposed, which is different from the only density dependent form

$$\zeta = \zeta_0 + \zeta_1 \frac{\dot{a}}{a} + \zeta_2 \frac{\ddot{a}}{\dot{a}} \tag{1}$$

It could be shown that it is equivalent to a modified equation of state(EOS)

$$p = (\gamma - 1)\rho + p_0 + w_H H + w_{H2} H^2 + w_{dH} \dot{H}. \tag{2}$$

And other interesting physical properties have been investigated in that kind of models [15] [16]. In this present paper, by largely extending its contents we will continue the study of a redshift dependent viscosity form $\zeta \propto \lambda_0 + \lambda_1 (1+z)^n$ as proposed in the previous work [17]. We concentrate on the late time singularity discussion and its entropy expression of the bulk viscosity DE model. For the phantom dark energy model, there exists a cosmic singularity in the future cosmos evolution, that is, the so-called cosmic "doomsday". As shown below, we could see that this viscosity DE model represents different evolution behaviors, and the future cosmic singularity will disappear under some proper parameter ranges selection. From this view of point, the model has the flexibility to produce either quintessence or phantom properties, that is, we can easily achieve its EoS either larger or less than characteristic -1.

An important tool for investigating dark energy model characters nowadays is by the introduction of some geometry quantities, for instance the statefinder diagnostic parameters [20], which are quantities dependent on high order derivatives of the scale factor, such as to the $\dddot{a}$. The usual statefinder parameter pair $\{r, s\}$ of the model concerned is calculated explicitly to demonstrate the viscosity DE model behaviors. In our previous work, deceleration parameter and Om diagnostic parameter [21] are performed to show the properties. We have found that in statefinder pair plane, our model and ΛCDM model could be easily discriminated in some redshift ranges. We prospect that increasing the quality and quantity of measurement data will enhance our ability to make accurate discrimination of different current cosmology models and rule out some. At the same time, new diagnostic methods merit further investigations, especially diagnostic quantity in the higher order perturbation level.

The paper is organized as follows: in the second section, some general features and remarks about viscosity dark energy model are given, and we further discuss the redshift dependent model. In the third section, we give the singularity discussion of our model. Some solutions are given there. In Sec. **IV**, we calculate entropy of this viscosity model. Finally, conclusion and discussions are presented. We leave the data fitting procedure in the appendix.

II. VISCOSITY DARK ENERGY MODEL

As well-known so far, the general theory of relativity for gravity and standard model for particle physics are very successful to describe the universe phenomena before the astrophysics exotic behaviors discovered, such as the roughly flat rotation curves for the spiral galaxies at large distance and the speed-up for the universe expansion at current stage evolution that are now commonly attributed as the cosmic "dark" physics evidences. The cosmic dark sector, often divided as mainly the dark energy and dark matter parts, consists of about 95% of the total cosmic ingredients and we know that purely gravitational probes are blind to the two main "flavors", that is the dark matter and dark energy can not be separated clearly. So at present evolution stage for our physical universe, a single unified dark fluid follows to describe the main cosmic media. In the Friedmann Robertson Walker metric the cosmic fluid is usually described by its homogenous and isotropic density ρ as well as the pressure p, that is, the pressure p can be divided into two additive parts (dark energy and matter) at least:

$$p = p_m + p_{de}. \tag{3}$$

Here we assume that other cosmic components, such as radiation and curvature contributions, are negligible as they may play un-important roles in our present description to the late universe evolution. It may be a simplifying view to treat cosmic fluid dividedly, but we consider that it is a more practical way for modeling dark energy and dark matter in a unified single fluid for the study of the late universe evolution. Also in our concrete viscosity DE model building, the usual assumption that cosmic fluid simply is perfect is not kept. Instead, we assume that the cosmic fluid for current universe evolution is better described by a single non-perfect fluid encoded with bulk viscosity effects. Therefor the modified pressure with viscosity term can be written directly as:

$$\tilde{p} = (\gamma - 1)p - 3\zeta H, \tag{4}$$

where ζ is the bulk viscosity parameter. It is useful to keep in mind that the relation between single fluid and multi-fluid framework, that is, in the multi-fluid case, the dark energy pressure p_{de} comes mainly from a bulk viscosity term added to the perfect fluid. This treatment could also be regarded as an effective method which revises the dark energy equation of state (EoS). If we set thermodynamics index $\gamma = 1$ in this paper, the cosmic pressure is mainly due to the viscous effects. Using the Friedman equation, we can get the revised equation of state for the relatively complex cosmic fluid:

$$\tilde{p} = -3\kappa\zeta\sqrt{\rho} \tag{5}$$

where in the equation above $\kappa^2 = \frac{8\pi G}{3}$.
If we define the effective EoS as usual $\omega = p/\rho$,then in this case:

$$\omega = -3\kappa\frac{\zeta}{\sqrt{\rho}} \tag{6}$$

With the aim to be consistence with current astrophysics observation results, the value of equation of state today should be

$$\omega|_0 \sim -1 \tag{7}$$

where the sub script zero indicates today's value, so that,

$$\zeta \sim \frac{H^2}{3\kappa^2} \tag{8}$$

If we fix the equation of state, that is, ω is a constant ω_0, we can have an interesting relation from eq.(6)

$$\rho = \frac{9\kappa^2\zeta^2}{\omega_0^2} \tag{9}$$

Three aspects on the viscosity DE model building are detailed below.

A. A General function for the Hubble parameter with viscosity contribution

For consistent with observation data, especially supernova data, we need calculate the integration of Hubble parameter, such as quantity like $\int \frac{1}{H(z)}\mathrm{d}z$, so to obtain more knowledge on $H(z)$ is certainly very useful. With the improved data quality and quantity of direct $H(z)$ parameter observations (like the Hubble Space Telescope project now running in the sky) we can use this better constrained Hubble rate in the fitting procedures to get more information on our observational universe evolution.

The conservation equation with a viscosity term is:

$$\dot{\rho} + 3H\rho = 9\zeta H^2. \tag{10}$$

With the Friedman equation, we can rewrite the conservation equation in terms of a Hubble parameter:

$$\dot{H} + \frac{3}{2}H^2 = \frac{3}{2}\zeta H. \tag{11}$$

Using the relation $\mathrm{d}t = \frac{1}{aH}\mathrm{d}a$, we can write the above formula as a differential equation to the scale factor:

$$\frac{\mathrm{d}H}{\mathrm{d}a} + \frac{3}{2a}H = \frac{3\zeta}{2a}. \tag{12}$$

Its form solution is thus obtained as a general integral function of the scale factor:

$$H = C_1 a^{-3/2} + \Big[\int \frac{3\zeta}{2a}\exp\big(\int \frac{3}{2a}\mathrm{d}a\big)\mathrm{d}a\Big]\exp\big(-\int \frac{3}{2a}\mathrm{d}a\big). \tag{13}$$

where the coefficient C_1 is an integral constant. For a different viscosity form $\zeta(a)$ given out, we could derive a concrete Hubble parameter expression accordingly. So we have obtained a general way to build the bulk viscosity dark energy models. We note that the emergent of the first term in eq. (13) which looks like the matter dominated contribution. By making the single fluid assumption, that is, the concrete ingredient of cosmic density is not specified, the $a^{-3/2}$ term naturally arises besides others, and contributes to a a^{-3} term in the function of H^2. Best fitting results with available observational data in previous work [17] is consistence with identifying the coefficient of this term as dark matter mass ratio $\Omega_m^{1/2}$.

B. The Redshift Dependent Viscosity

When the bulk viscosity parameter is specified, we could discuss cosmology with the unified dark energy evolution behaviors. Variable viscosity parameter like the density dependent model $\zeta(\rho) = \alpha\rho^m$ has been investigated extensively as in ref. [22]. In the previous work, we have proposed a new analytical form of viscosity parameter in the flat FRW space-time:

$$9\lambda = \lambda_0 + \lambda_1(1+z)^n, \tag{14}$$

where the viscosity is re-written as $\lambda = H_0\zeta/\rho_{cr0}$, and z is the redshift. The exponent parameter n is to be best fitted by observational data sets, so as the λ_0 and λ_1 two arbitrary constants.From Friedmann equation containing the viscosity term we have:

$$\frac{\ddot{a}}{a} = -\frac{\kappa}{2}(\rho + 3\tilde{p}), \tag{15}$$

that is exactly in detail

$$\frac{\ddot{a}}{a} = -\frac{\kappa}{2}(\frac{\dot{a}}{a})^2 + \frac{3}{2}\kappa^2\zeta\frac{\dot{a}}{a}. \tag{16}$$

We could derive the function of Hubble parameter H evolution with the redshift as:

$$\frac{H}{H_0} = \lambda_2(1+z)^{1.5} - \frac{\lambda_1}{2n-3}(1+z)^n + \frac{\lambda_0}{3}, \tag{17}$$

The coefficient λ_2 is an integration constant and constrained self-consistently by the above relation when taking z=0. We use this function of Hubble parameter by fitting the latest released Constitution supernova data sets to get these free constant parameters. By the numerical fitting processes with statistic analysis that is compiled in the appendix of this work, and choose the parameter $n = -1$ as the best value, we could get a result closed to the ΛCDM model when comparing the deceleration parameter derived from different models with various n values. In the FIG. 1, the theoretical distance modulus curve of this redshift dependent viscosity DE model is plotted, which gives an acceptable fitting results with the latest released supernova data sets.

C. Statefinder diagnostic analysis

Two statefinder parameters are defined in terms of the scale factor and Hubble parameter by

$$r = \frac{\dddot{a}}{aH^3}, \qquad s = \frac{r-1}{3(q-\frac{1}{2})}, \tag{18}$$

where the deceleration parameter q is defined as $q = -\frac{\ddot{a}}{aH^2}$.

If we have already obtained the function form of Hubble parameter $H(z)$, it is more convenient to express the diagnostic statefinder parameter pair $\{r, s\}$ in terms of $H(z)$, therefore $r(z)$ and $q(z)$ become

$$r = (1+z)q' - q[1 - 2(1+z)\frac{H'}{H}], \tag{19}$$

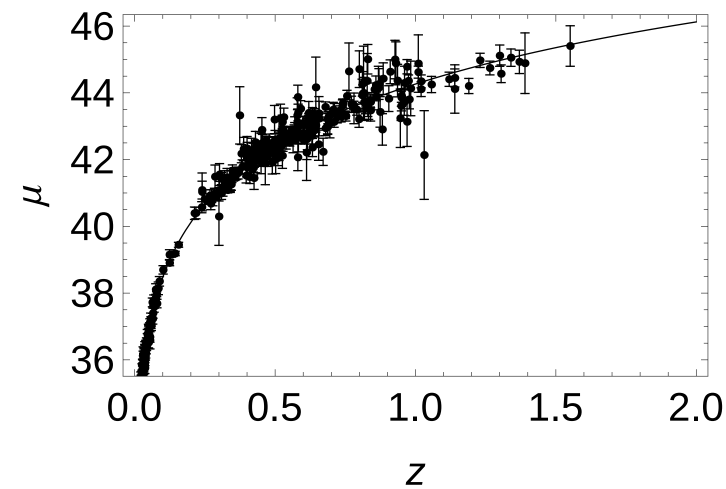

FIG. 1: The relation between distance modulus and redshift. The solid line corresponds to the theoretical curve of our viscosity DE model and the dots with error bar are the observational data.

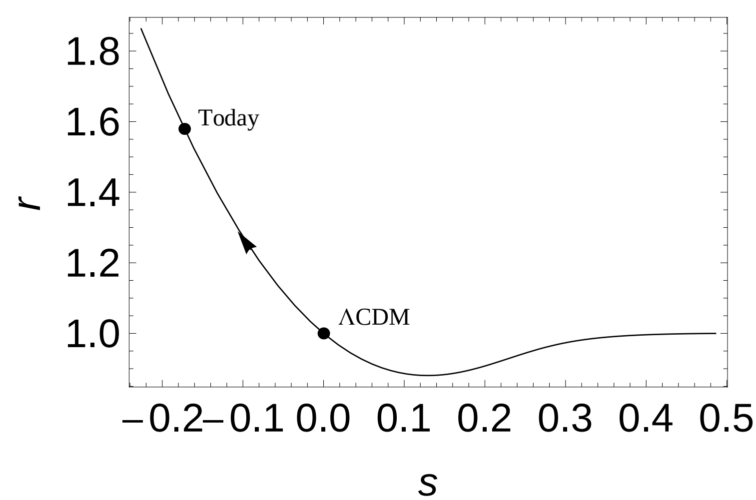

FIG. 2: The universe evolution is diagnostic by the evolution trajectory in the $r - s$ plane for the $n = -1$ condition. The fixed point at (0,1) corresponds to the ΛCDM model.

$$q = (1+z)\frac{H'}{H} - 1, \tag{20}$$

where the prime represents derivative with respect to the redshift z.

Functional relation between the diagnostic parameters r and s is displayed above. In the $r - s$ plane, the fixed dot at $\{0, 1\}$ corresponds to the concordant ΛCDM model. Here, we only consider the special case in which the parameter $n = -1$ for the viscosity DE model and other parameters λ_1, and λ_2 have been best fitted in this case.

The universe evolution curve by the statefinder dialogistic parameter passes the point which corresponds to the ΛCDM model. For our model with the viscosity effects in the best fitting case, this point is just at when the universe evolves at redshift $z \simeq 0.447$ where the statefinder diagnostic

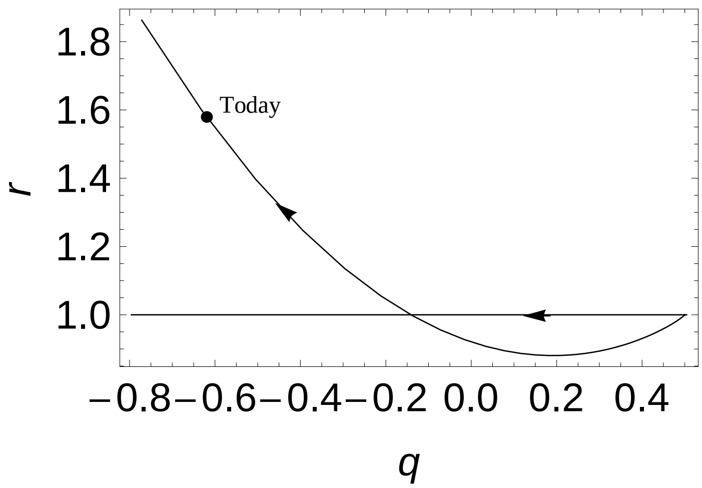

FIG. 3: Evolution trajectory in $r-q$ plane to demonstrate the universe evolution in another diagram view.

parameters are $\{s=0, r=1\}$ by numerical calculations. We conclude that the statefinder diagnostic analysis shows model degeneracies (the ΛCDM model and our viscosity DE model) at this special point, but it could be easily to discriminate between viscosity DE models and the ΛCDM model elsewhere. The statefinder diagnostic parameter values today with the redshift z=0 correspond to the point with $r=1.579$ and $s=-0.173$. Its location is represented in the figure 2, too.

To illustrate more details for the unified viscosity DE model different from the ΛCDM model we also plot the parameter r(q) function trajectory in the $r-q$ plane as shown in the figure 3. The black arrow points towards the direction the universe evolves to and the horizonal line corresponds to the ΛCDM model evolution. We know that the negative geometric quantity deceleration parameter q represents that the universe expansion is undergoing an acceleration stage, hence it could be obviously shown that the evolution tendency trajectory from the cosmic expansion deceleration range to expansion acceleration range, which corresponds to our universe evolution from the right to the left side along the horizonal q axis in the $q-r$ plate.

To demonstrate further the similarities and differences between the unified viscosity DE model and the ΛCDM model we plot and compare the q-z relations of the two models as shown in the figure 4, too, which provides another view to compare the two models.

III. THE SCALE FACTOR AND THE FINITE FUTURE SINGULARITY

It is well known that the phantom dark energy models with a negative equation of state have had exotic cosmic evolution behaviors[23]. Cosmos evolution may be driven to a singularity in the finite future, such as the so called Sudden, Big-rip, or Big-brake singularities[24] [25]. And as for the newly explored type of singularity named Barotropic index w-singularities [26], it could be led

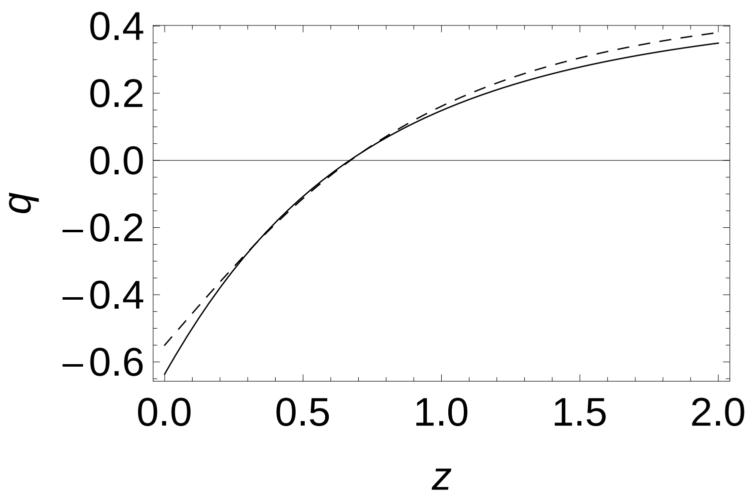

FIG. 4: The $q - z$ relation diagram. The dashed line corresponds to the deceleration parameter relation with the redshfit computed from the ΛCDM model with the model parameter $\Omega_m = 0.3$, while the solid line delineates the best fitted viscosity DE model discussed here.

by a scale factor with a typical expression:

$$a(t) = A + B\left(\frac{t}{t_s}\right)^{\frac{2}{3\gamma}} + C\left(D - \frac{t}{t_s}\right)^n. \tag{21}$$

where the parameter t_s indicates a finite future time when the singularity will occur and the A, B, C, D, n and γ are all free parameters to be determined. Singularities are harmful to the prediction power for a theory and theoretical model buildings. With the Hubble parameter got previously, we could discuss more details on the evolution properties, especially the future singularity of our viscosity DE model. Since the scale factor $a(t)$ can be expressed in terms of the redshift as $\frac{1}{1+z}$ when we set $a(0) = 1$, the Hubble parameter can thus be rewritten as:

$$\frac{H}{H_0} = \lambda_2 a^{-1.5} - \frac{\lambda_1}{2n-3}a^{-n} + \frac{\lambda_0}{3}. \tag{22}$$

As we can easily see above that the unified viscosity DE model behaves generally as the ΛCDM model in the early universe evolution, especially when n negative, we will discuss several special cases to the above general evolution features of eq.(22)

(a) the $n < 0$ case

with the increasing of the scale factor, the first term, proportional to $a^{-1.5}$, will decay to zero, while the n-power term will increase and dominate in the future universe evolution. The Hubble parameter will be approximated then as:

$$\frac{H}{H_0} \simeq -\frac{\lambda_1}{2n-3}a^{-n} + \frac{\lambda_0}{3}. \tag{23}$$

For the $n = -1$ case, in which we have a better comparison with the ΛCDM model. The differential equation for the scale factor evolution is:

$$\dot{a} \simeq \frac{\lambda_1 H_0}{5} a^2 + \frac{\lambda_0 H_0}{3} a. \tag{24}$$

We could get an approximate solution for the scale factor evolution behaviors:

$$a(t) = \frac{5\lambda_0}{3\lambda_1} \frac{\exp(\frac{\lambda_0 H_0}{3} t - \frac{\lambda_0 H_0}{3} t_s)}{1 - \exp(\frac{\lambda_0 H_0}{3} t - \frac{\lambda_0 H_0}{3} t_s)}, \tag{25}$$

where t_s is the time parameter when a future singularity happens (when $t = t_s$, the scale factor has a finite future singularity: $a \to \infty$).

As we all know, the scale factor $a(t)$ is a basic and crucial quantity to our understanding of the cosmic evolution history. We may use this approximate solution of the scale factor in the future evolution to discuss more details of this viscosity DE model near future singularity. The cosmic media density reads as:

$$\rho = \frac{3H^2}{\kappa}. \tag{26}$$

Inserting the approximated expression of Hubble parameter above into the above expression we can have:

$$\rho = \frac{3}{\kappa}\Big(\frac{\lambda_1 H_0}{5} a + \frac{\lambda_0}{3}\Big)^2. \tag{27}$$

We could directly see that if $a \to \infty$ as $t \to t_s$, the cosmic density will diverge, too.

By the Eq.(5), we could also see that the cosmic pressure $|p| \to \infty$ as $a \to \infty$. From the EoS defined above we obtain:

$$\omega = -3\kappa \frac{\zeta}{\sqrt{\rho}}, \tag{28}$$

we have $\omega \to \omega_s$ where the ω_s is the EoS when $a \to \infty$. In the ref. [25], authors classify the future singularities in the following way:

(i) Type **I**("Big Rip"): For $t \to t_s$, $a \to \infty$, $\rho \to \infty$, and $|p| \to \infty$,

(ii) Type **II**("Sudden"): For $t \to t_s$, $a \to a_s$, $\rho \to \rho_s$, and $|p| \to \infty$,

(iii) Type **III**: For $t \to t_s$, $a \to a_s$, $\rho \to \infty$, and $|p| \to \infty$,

(iv) Type **IV**: For $t \to t_s$, $a \to a_s$, $\rho \to 0$, $|p| \to 0$, and higher derivatives of H diverge.

We could see that the model we discussed above falls basically into the Type **I** category , that is, the so-called "Big Rip" singularity. Another relevant problem to solve is the determination of singularity time t_s. In the discussions above, the parameter t_s emerges as an integration constant.

When t increases to some finite value, cosmos will come to the singularity, and we can choose this value as the cosmic singularity time t_s. Some authors [27] have given an approximation value of a cosmic “doomsday”, as $t_s = 35$ Gyr. As for the early universes evolution in our unified viscosity DE model it returns to the Friedmann phase or de Sitter phase.

(b) The $n = 0$ case

In this condition, the viscosity parameter will became a constant $\zeta = \frac{\rho_{cr0}}{H_0}(\lambda_0 + \lambda_1)$. Since $h(z = 0) = 1$ in the special case, it is easy to verify that $\zeta > 0$. Hence, the scale factor solution to the Friedmann equation Eq.(16) is

$$a(t) = k\big[1 + (1+m)\mathrm{exp}(\frac{3}{2}\kappa^2\zeta, t)\big]^{\frac{1}{1-m}} \tag{29}$$

where k is a normalization factor, and $m = \frac{\kappa}{2}$.

And in this case, there is no the finite time singularity. A de Sitter phase of universe will emerge as $t \to \infty$. A parallel remark could be made here that for the $n < 0$ case, we can neglect the $\lambda_1(1+z)^n$ term from the viscosity contributions. Then we have obtained a solution with the same form as Eq. (29). This is an inflation-like solution, but when $t = 0$, $a(init) \neq 0$, there is a initial condition uncertainty problem.

(c) The $n > 0$ case

Terms about scale factor in Hubble parameter will decay and vanish, so as $t \to \infty$, the Hubble parameter will become a constant. Then, the term $\frac{\lambda_0}{3}$ will play a role as the effective cosmological constant which dominates the late universe evolution, therefore the universe will enter into a de Sitter phase.

IV. ENTROPY AND AN ARROW OF TIME

Entropy is related causality and the second law of thermodynamics is usually regarded as the major physical manifestation of the arrow of time, from which many interesting consequences can be derived. In this section, we will discuss the entropy expression in our unified viscosity DE model. Due to the non-perfect fluid property assumption of the viscosity DE model, the entropy will change in contrast with the case for perfect fluid models, in which $\frac{\mathrm{d}S}{\mathrm{d}t} = 0$ (Where we define S as the entropy of the system in unit volume).

In Refs. [28, 29, 30, 31], the general formula for the entropy expression has been given as:

$$S^\mu_{;\mu} = \frac{\eta}{T}\sigma_{\mu\nu}\sigma^{\mu\nu} + \frac{\zeta}{T}\theta^2 + \frac{1}{\kappa T}Q_\mu Q^\mu, \tag{30}$$

where the S^μ is the entropy four-vector, η the shear viscosity, T the system temperature, $\sigma_{\mu\nu}$ the shear tensor, θ the expansion factor, κ the thermal conductivity and the four-vector Q^μ the space-like heat flux density. And the entropy four-vector S^μ is defined by:

$$S^\mu = \delta S U^\mu + \frac{1}{T}Q^\mu, \tag{31}$$

where δS is the ordinary entropy per unit volume in single fluid, and the contributions from different ingredients are not specified. In the universe evolution system the expansion tensor $\theta_{\mu\nu}$ is defined as

$$\theta_{\mu\nu} = \frac{1}{2}(U_{\mu;\alpha}h^{\alpha}_{\nu} + U_{\nu;\alpha}h^{\alpha}_{\mu}). \tag{32}$$

The scalar expansion factor thus is $\theta = \theta^{\mu}_{\nu} = U^{\mu}_{;\mu}$, especially in the FRW background $\theta = 3H$. The shear tensor is defined as $\sigma_{\mu\nu} = \theta_{\mu\nu} - \frac{1}{3}h_{\mu\nu}\theta$, where $h_{\mu\nu}$ is defined by $h_{\mu\nu} = g_{\mu\nu} + U_{\mu}U_{\nu}$. Defining the four-acceleration of the fluid as $A_{\mu} = \dot{U}_{\mu} = U^{\nu}U_{\mu;\nu}$, then the space-like heat flux density four-vector is given by

$$Q^{\mu} = -\kappa h^{\mu\nu}(T_{,\nu} + TA_{\nu}). \tag{33}$$

If the system concerned is in the thermal equilibrium, therefore the four-vector $Q^{\mu} = 0$.
For the FRW background with thermal equilibrium condition satisfied, we can have

$$\sigma_{\mu\nu} = 0, \qquad \theta = 3H. \tag{34}$$

And the Eq. (30) reduces to

$$S^{\mu}_{;\mu} = \frac{\zeta}{T}\theta^2, \qquad S^0 = \delta S, \qquad S^i = 0 \tag{35}$$

Hence, we get the differential equation for the entropy expression with the argument as cosmic time below

$$S^0_{,0} + 3HS^0 = 9\frac{\zeta}{T}H^2 \tag{36}$$

We therefore can convert the argument to the scale factor $a(t)$, by using the relation $\frac{\mathrm{d}a}{\mathrm{d}t} = aH$. Here we make an assumption that the ambient temperature has the form that $T \propto a^{-\gamma}$, or $Ta^{\gamma} = T_0$ with a free scaling parameter γ, where T_0 is the temperature now:

$$\frac{\mathrm{d}S^0}{\mathrm{d}a} + 3\frac{S^0}{a} = 9\frac{\zeta}{T_0}\frac{\dot{a}}{a}a^{\gamma-1} \tag{37}$$

With already obtained relation as in the eq.(17):

$$\frac{\dot{a}}{a} = \lambda_2 a^{-1.5} - \frac{\lambda_1}{2n-3}a^{-n} + \frac{\lambda_0}{3}. \tag{38}$$

and with the definition $\zeta = \frac{\lambda\rho_{cr0}}{H_0}$ and $\lambda = \frac{\lambda_0}{9} + \frac{\lambda_1}{9}a^{-n}$, we could write the differential equation as

$$\frac{\mathrm{d}S^0}{\mathrm{d}a} + 3\frac{S^0}{a} = (\alpha + \beta a^{-n})(\lambda_2 a^{-1.5} - \frac{\lambda_1}{2n-3}a^{-n} + \frac{\lambda_0}{3})a^{\gamma-1}, \tag{39}$$

where the parameters $\alpha = \frac{\rho_{cr0}\lambda_0}{T_0}$ and $\beta = \frac{\rho_{cr0}\lambda_1}{T_0}$.

We work out the differential equation and find the general entropy expression as below

$$\begin{aligned} S^0 \;=\; & \frac{\alpha\lambda_2}{\gamma+1.5}a^{\gamma-1.5} - [\frac{\beta\lambda_0}{3(\gamma-n+3)} - \frac{\alpha\lambda_1}{(2n-3)(\gamma-n+3)}]a^{\gamma-n} + \\ & + \frac{\alpha\lambda_0}{3(\gamma+3)}a^{\gamma} + \frac{\beta}{\gamma-n+1.5}a^{\gamma-n-1.5} - \\ & - \frac{\beta\lambda_1}{(2n-3)(\gamma-2n+3)}a^{\gamma-2n} + ca^{-3}. \end{aligned} \tag{40}$$

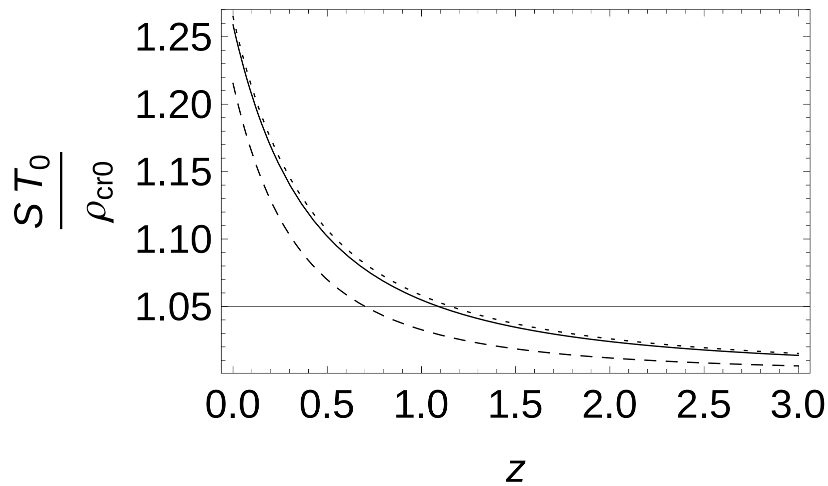

FIG. 5: The relation between the total entropy aS^0 of the universe in the unified viscosity DE model and the corresponding redshift. The dashed, solid and dotted lines correspond to the temperature parameter $\gamma = 0.5, 0.05$ and 0, respectively. The total entropy is increasing with the cosmic time flying (the corresponding redshift becoming smaller), and this monotonous increasing property can be regarded as an arrow of time pointing to the universe evolution direction.

For the special case with the $n = -1$, we can directly obtain

$$S^0_{n=-1} = \frac{\alpha\lambda_2}{\gamma+1.5}a^{\gamma-1.5}+[\frac{\beta\lambda_0}{3(\gamma+4)}+\frac{\alpha\lambda_1}{5(\gamma+4)}]a^{\gamma+1}+\frac{\alpha\lambda_0}{3(3+\gamma)}a^{\gamma}+\frac{\beta}{\gamma+2.5}a^{\gamma-0.5}+\frac{\beta\lambda_1}{5(\gamma+5)}a^{\gamma+2}+ca^{-3}. \tag{41}$$

We plot the evolution trajectory of the total entropy calculated above in the $S-z$ plane with dimension re-arranged. It is obvious to see that the total system entropy is increasing with the cosmic time flying, and this monotonous increasing property represents an arrow of time for describing the universe evolution. Though the quantum version of an arrow of time is still hotly debating the thermodynamic entropy may be a good depiction to sketch the whole or global universe evolution with very complicated ingredients inside.

V. SUMMARY AND DISCUSSIONS

In this letter we continue and largely extended our previous work on the single and unified bulk viscous fluid as a potential dark energy candidate by presenting an explicit viscosity form to mimic dark energy behaviors and confront it with current observational data sets. The specific feature here is a variable coefficient for the new bulk viscosity form proposed, characterized by two free parameters that can be best fitted by astrophysics observational data sets. the best fitting results have shown that this concrete model could yield theoretical prediction values in an acceptable level by working out the numerical processing to the latest released joint observational data sets.

Furthermore, we have performed the statefinder diagnostic parameter analysis to this unified viscosity DE model, finding that in most evolution stages of the unverse, statefinder parameters could be used to obviously distinguish this viscosity DE model from the ΛCDM model. But as shown in

the figure of the statefinder pair parameter $\{r, s\}$, the viscosity DE model evolution trajectory passes the special point corresponding to the well known ΛCDM model, where the models degeneration emerges. The new Om parameter diagnostics made previously could be a more powerful tool then, which might discriminate concrete models from the ΛCDM model in the whole evolution history.

In this present work, we particularly concentrate on the singularity behavior of the unified viscosity dark energy model. We find that different parameter range selection, especially the region of power parameter n, influences the finite future singularity. For the $n = 0$ case, which corresponds to the case with a constant viscosity, there is an exact solution for the evolution scale factor. For the early universe and the $n < 0$ case, the solution of the scale factor $a(t)$ has possessed the same structure. A further study of viscosity effects on the early universe evolution, especially during inflation stage, seems very interesting with rich possibilities.

In the context of the unified viscosity DE cosmology, we also calculate the entropy for the total evolution universe, which has been expressed in a complex form with this non-perfect viscosity media. Calculations of the total entropy for the viscosity universe evolution represent that the general second law of thermodynamics holds in the whole or global universe description. The worked out results show that in this complicated context, the total entropy is increasing with the redshift decreasing, or cosmic time flying, including the cosmic expansion accelerating stage as observational data indicating now, which has been plotted in figure 5 with three free parameter values chosen. The monotonous increasing property of the total entropy with cosmic time flowing, the viscosity DE universe evolution may provide us an arrow of time to describe the complex universe changing direction. Though a definite concept for the arrow of time is debating, now very puzzling in different situations or versions: classical physics, quantum physics, cosmology and quantum gravity, especially it is essential for a correct quantum gravity theory to be expected to appear, the thermodynamics second law is generally believed to hold to describe the global evolution of our observational universe. So the reasonable entropy expression is richly encoded helpful information for the concerned system. It is certainly interesting and worthy of further efforts.

Dark energy physics involves many fundamental concepts and beyond in our already established "standard models" for both particle physics and gravity. Viscosity media seems to relate the matter sector to the geometric gravity side via the Einstein's general relativity equation. Actually it also can be reconstructed effectively from the left side to the right side of the equation by modified gravity or extra dimension models for example, too, which is also intriguing. With the available and upcoming high quality and increasingly large amount of astrophysics observational data, especially the good low redshift SNe Ia data sets with less uncertainty for possible errors from the dust effects alleviated under control we expect the ten-years old mysterious dark energy problem will be pinned down not too long compared with the long time standing unsolved dark matter mystery. Maybe the practically unified viscosity DE model or the like can provide an economic mechanism to answer the both uncovered secrets in one dark sector, a tale for the two mysteries.

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Appendix

We briefly summarize the joint statistical data analysis method here by numerical processings to confront theoretical models with the currently observational data sets. Supernova type 1a data provides the direct evidence of cosmic accelerating expansion. Using the χ^2 statistics method, DE cosmological model parameters could be best fitted. Recently, a large sample of low redshift($z < 0.08$) has been released freely[32]. Complied with the Union sample[33], the largest data set which could be obtained now openly, is the Constitution data [34] with the newly high quality low redshift data sets included. In this paper, we perform the joint data fitting processings with this new compiled data set. The observations of supernovas measure essentially the apparent magnitude m, which is related to the luminosity distance d_L by

$$m = M + 5\log_{10} D_L\left(z\right), \tag{42}$$

where the distance $D_L\left(z\right) \equiv \left(H_0\right) d_L\left(z\right)$ is the dimensionless luminosity and

$$d_L = (1+z) d_M(z), \tag{43}$$

where d_M is the co-moving distance given by

$$d_M = \int_0^z \frac{1}{H(z')} dz'. \tag{44}$$

Also,

$$\mathcal{M} = M + 5\log_{10}\left(\frac{1/H_0}{1Mpc}\right) + 25, \tag{45}$$

where M is the absolute magnitude which is believed to be constant for all supernova of type Ia. The data points in these samples are given in terms of the distance modulus

$$\mu_{obs} \equiv m(z) - M_{obs}(z). \tag{46}$$

We employ it for doing the standard statistic analysis. So the χ^2 is calculated from

$$\chi^2 = \sum_{i=1}^{n} \left[\frac{\mu_{obs}(z_i) - \mu_{th}(z_i; c_\alpha)}{\sigma_{obs}(z_i)}\right]^2. \tag{47}$$

Theoretical calculation value is expressed as μ_{th}, and $\mu_{th} = M' - 5\log_{10} D_{Lth}(z_i; c_\alpha)$, where $M' = \mathcal{M} - M_{obs}$ is a free parameter need to fit as well and $D_{Lth}(z_i; c_\alpha)$ is the theoretical prediction for the dimensionless luminosity distance of a supernovae at a particular distance, for a concreste model with some parameters c_α. The aid of this procedure is to determine the value of c_α.

On the other hand, the shift parameter $\mathcal{R}$ inferred from CMB power spectrum of WMAP year five data and the distance parameter $\mathcal{A}$ from the BAO data of LSS, like SDSS and 2dF, are considered to give effective contributions to the joint statistical data fitting. The shift parameter $\mathcal{R}$ is defined in refs.[36] and [37] as

$$\mathcal{R} \equiv \sqrt{\Omega_m} \int_0^{z_*} \frac{dz'}{h(z')}, \tag{48}$$

and WMAP5 results [35] have updated the redshift of recombination to be at $z_* = 1090$. Its detail meaning can be found in reference [38]. The distance parameter $\mathcal{A}$ is given by

$$\mathcal{A} \equiv \sqrt{\Omega_m}\, h(z_b)^{-\frac{1}{3}} \left(\frac{1}{z_b} \int_0^{z_b} \frac{dz'}{h(z')}\right)^{\frac{2}{3}}, \tag{49}$$

where commonly value $z_b = 0.35$ is used.

Jointly taking into considering parameters $\mathcal{R}$ and $\mathcal{A}$, we can soundly use the total χ^2_{total} to make the best fitting analysis:

$$\chi^2_{total} = \chi^2 + \left(\frac{\mathcal{R} - \mathcal{R}_{obs}}{\sigma_{\mathcal{R}}}\right)^2 + \left(\frac{\mathcal{A} - \mathcal{A}_{obs}}{\sigma_{\mathcal{A}}}\right)^2. \tag{50}$$

So far this is the most acceptable joint statistical data analysis method for exploring the DE mystery and for the DE cosmology model buildings to confront with the astrophysics observational data sets obtained. Complementary with the increasing amount and accuracy of lensing, cluster survey and others data sets to be obtained we are confident it will be not far away to pin down the relative young dark energy and long standing dark matter identities, and eventually to solve the dark sector puzzles with possible new discoveries.

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